**FIELLER’S THEOREM**

**Theory and Application With Computations in SAS**

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**FIELLER’S THEOREM -Theory and Application With Computations in SAS**

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**Introduction**

Confidence intervals for ratio estimators can be computed using Fieller’s Theorem. The enclosed two macros written in SAS compute a (1- ) x 100% confidence interval for such estimators.

## Theory

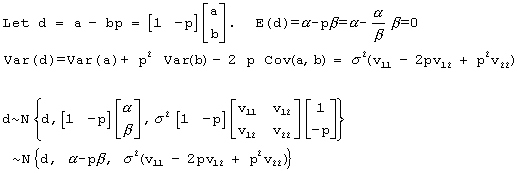
The theory behind Fieller’s fiducial intervals as presented by Dr.

George Milliken of Kansas State University and Dr. John Miller of George Mason University is as follows. Consider the ratio  and its estimator



m=a/b where follows a bivariate normal distribution with mean and

covariance .



Now form a t statistic since s unknown.

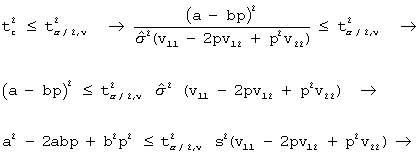
i

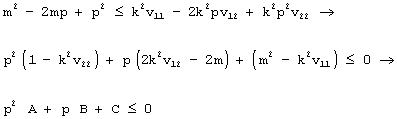
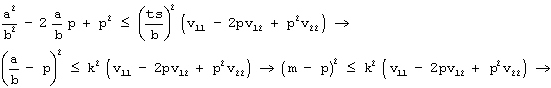


The (1-) x 100% confidence interval about p is that set of values of p such that

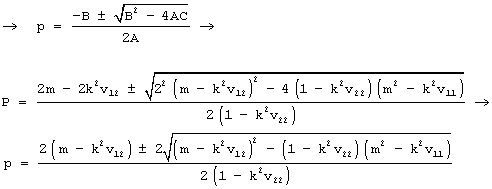


I need to find all p such that





Next solve

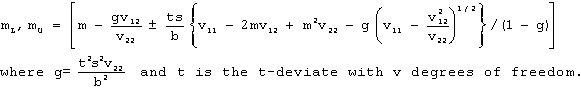


Fieller’s theorem states that upper and lower fiducial limits to p are

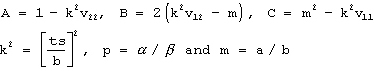


The computations found in the popular text “Statistical Method in Biological Assay” by Finney are similar. In this text application of

Fieller’s theorem computes the upper and lower fiducial limits to p as



Note that in the above calculations that



By rewriting the equation for k, I can find an expression for t. Then compute the area corresponding to the minimum p-value for which an appropriate solution is possible.

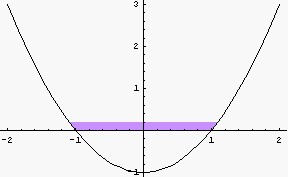


Now compute 2( 1-T( kb/s) ) where T is the cdf of a t random variable.

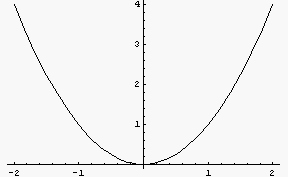
This is the minimum observed significance level which will yield an appropriate interval.

Four possible situations can occur in the solution of these equations.

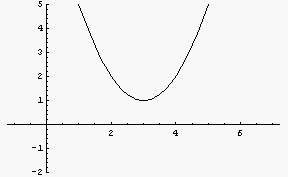
Case 1a. A > 0 and B²-4 A C > 0 Two distinct solutions exist.



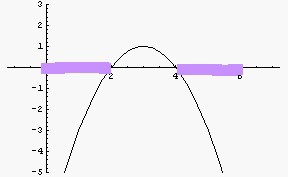
Case 1b. A > 0 and B²-4 A C = 0 Single solution(a single point) exists.



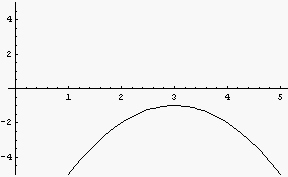
Case 2 A > 0 and B²-4 A C < 0 A confidence interval does not exist.



Case 3 A < 0 and B²-4 A C > 0. Inequality  does not lead to a useful confidence region.



Case 4 A < 0 and B²-4 A C < 0. Confidence interval does not exit (may as well not collected any data at all).



All 4 situations are reported in the SAS macros. Details are presented under title “Interval Diagnostics”.

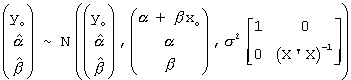
## Application.

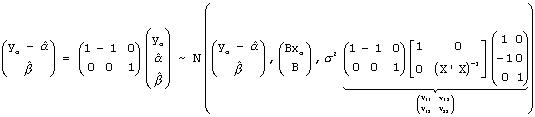
This approach requires a simple linear regression equation of the form y = a x + b where follows a bivariate normal distribution. Suppose

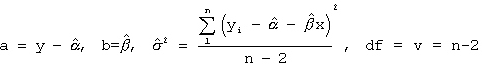
further that a and b are unbiased estimates of and  forming a linear function of a set of observations with normally distributed errors. An ANOVA of the data will give a error mean square, s², with v degrees of freedom. The estimated variances of a, b, and their covariance may be expressed as s²v11, s²v22 and s²v12.

The technique is used in calibration where the unknown is x. y is given by the experimenter and using the formula , x is computed for various values of y.

First the joint distribution of is found.



Computation of a confidence interval for the ratio can be simplified using matrix operations.

Now apply Fieller’s theorem with

## Sample Data Set

**(Graybill (1976), Theory and Application of the Linear Model, page 274)**

x y

|  |  |
| --- | --- |
| . | 2.10 |
| 1.0 | 4.81 |
| 1.1 | 3.60 |
| 1.3 | 4.90 |
| 1.6 | 3.05 |
| 1.8 | 3.44 |
| 1.8 | 3.17 |
| 1.8 | 3.34 |
| 2.1 | 1.61 |
| 2.4 | 1.22 |
| 2.6 | 0.20 |
| 2.6 | 1.56 |
| 2.7 | 0.55 |
| 2.9 | -2.56 |
| 3.0 | -0.34 |
| 3.5 | -2.56 |
| 3.6 | -2.96 |
| 4.1 | -1.04 |
| 5.2 | -4.64 |

## Simple Linear Regression using sample data set.

Analysis of Variance

Parameter Estimates

Covariance of Estimates

CALIBRATION using SAS REGINV Macro

Value of G Should be Less than 0.1

Value of

Dependent

Lower

Predicted

Value of Independent

Upper

Value of

Standard

Error of

Critical Value

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | DF | Sum of  Squares | Mean  Square | F Value | Pr > F |
| Model | 1 | 119.36728 | 119.36728 | 107.25 | <.0001 |
| Error | 16 | 17.80815 | 1.11301 |  |  |
| Corrected Total | 17 | 137.17543 |  |  |  |
| Root MSE |  | 1.05499 | R-Square | 0.8702 | |
| Dependent | Mean | 0.96389 | Adj R-Sq | 0.8621 | |
| Coeff Var |  | 109.45167 |  |  | |

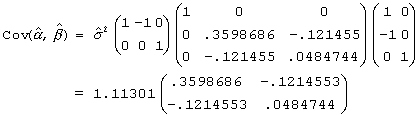
|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Variable | DF | Parameter  Estimate | Standard  Error | t Value | Pr | > |t| |
| Intercept | 1 | 6.99091 | 0.63288 | 11.05 |  | <.0001 |
| x | 1 | -2.40546 | 0.23228 | -10.36 |  | <.0001 |

|  |  |  |
| --- | --- | --- |
| Variable | Intercept | x |
| Intercept | 0.4005369955 | -0.135180861 |
| x | -0.135180861 | 0.0539524502 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Variable | Bound | Variable | Bound | Alpha |
| 2.1 | 1.07314 | 2.03325 | 2.99336 | 0.05 |

|  |  |  |
| --- | --- | --- |
| Independent  Variable | from  t-distribution | g |
| 0.45290 | 2.11991 | 0.041903 |

Using a contrast matrix compute the covariance matrix of the joint density of .



FiellerF

The Minimum Alpha (observed significance level) is 1.6845455E-8

Lower Upper

a b df alpha g Bound ratio Bound

-4.89091 -2.40546 16 0.05 0.041903 1.03150 2.03325 2.99369



Output from Fieller macros written compute a 95% confidence interval of (1.03,2.99). Actual output follows:

FiellerM

The Minimum Alpha (observed significance level) is 1.6845455E-8

Interval Details

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | | | Critical Value  from | Lower |  | Upper |
| a | b | df alpha | t-distribution | Bound | ratio | Bound |
| -4.89091 | -2.40546 | 16 0.05 | 2.11991 | 1.03150 | 2.03325 | 2.99369 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| msg |  |  | anum | bnum | v11 | v12 | v22 | v |
| Assumptions | are | Met | -4.89091 | -2.40546 | 1.35987 | 0.12146 | 0.048474 | 16 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| alpha | s | | stat0 | | stat | coef | df | t | m | k |
| 0.05 | 1.05499 | | 10.3561 | | 1.6845E-8 | 0.975 | 16 | 2.11991 | 2.03325 | -0.92975 |
| 1-k^2\* | |  | |  |  |  | | Under |  |  |
| v22 | | A | | AAI | B | C | | Radical | LB | UB |
| 0.95810 | | 0.95810 | | 0 | -3.85652 | 2.95859 | | 3.53428 | 1.03150 | 2.99369 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| a | b | | Interval  m | Details  s | v11 | v12 |
| -4.89091 | -2.40546 | | 2.03325 | 1.05499 | 1.35987 | 0.12146 |
| v22 | df | code\_ disc | code\_g | LB | UB | |
| 0.048474 | 16 | YES | YES | 1.03150 | 2.99369 | |

## SAS Macros

REGINV written by SAS Institute allows for more significant digits since it uses output from proc glm in all of its computations. However, REGINV only applies to the specific case of calibration of the independent variable in a simple linear regression and hence is not as general in its application.

Two new macros were written to implement the use of Fieller’s Theorem. They are more generic than the SAS macro reginv referenced above in that the code performs ratio estimation where the numerator and denominator follow a bivariate normal distribution. Inputs consist of the ratio, the covariance matrix, the mean square error and desired significance level. Diagnostics are given for cases where an acceptable interval does not exist. A slight difference in interval endpoints can be attributed to roundoff error.

The first macro is labeled FiellerM.sas (M=Milliken) and the second is called FiellerF.sas (F=Finney). While the formulas are slightly different both macros compute the same interval. The second macro is included for completeness since Finney’s methodology is often referenced in the literature. A modified version of REGINV is included as reginv2.

The program Graybill.8.10.sas calls the two macros. All code was written using SAS versions 7 and 8 on a PC with Microsoft Windows 98 operating system. They are included as an attachment to this paper.

## Acknowledgements

Draper and Smith (1998), Applied Regression Analysis -Third Edition, John Wiley & Sons, page 85 (3.28)

Finney, David J. (1978), Statistical Method in Biological Assay -Third Edition, Charles Griffin Company LTD

Graybill, Franklin A.(1976), Theory and Application of the Linear Model, Wadsworth & Brooks/Cole, Pacific Grove, California, Confidence Interval Formula is given on page 335

Miller, John Ph.D., George Mason University, Stat 656 Regression Analysis course notes

Milliken, George, Ph.D., Kansas State University, STAT 861 Linear Models II course notes, pages 8-34 to 8-37

SAS Principles of Regression Course Notes, Macro to produce inverse confidence intervals for individual values is found on pages 639-640, SAS Institute, Inc.: Cary, North Carolina

# SAS Source Code

**Graybill.8.10.sas** Program which calls the macros.

/\* SAS and FIELLER'S THEOREM, Graybill Problem 8.10 \*/

options nodate number center linesize=76 pagesize=54 pageno=1; options replace probsig=1 firstobs=1 obs=max;

options mrecall label;

options xsync noxwait yearcutoff=1950; title;

ods listing;

data Fieller; input x y @@; datalines;

|  |  |
| --- | --- |
| . | 2.10 |
| 1.0 | 4.81 |
| 1.1 | 3.60 |
| 1.3 | 4.90 |
| 1.6 | 3.05 |
| 1.8 | 3.44 |
| 1.8 | 3.17 |
| 1.8 | 3.34 |
| 2.1 | 1.61 |
| 2.4 | 1.22 |
| 2.6 | 0.20 |
| 2.6 | 1.56 |
| 2.7 | 0.55 |
| 2.9 | -2.56 |
| 3.0 | -0.34 |
| 3.5 | -2.56 |
| 3.6 | -2.96 |
| 4.1 | -1.04 |
| 5.2 | -4.64 |
| ;  data | Fieller; set Fieller; |
| proc | print; var x y; run; |
| proc | glm data=Fieller; |

model y = x / xpx inverse ss3;

ods output ParameterEstimates=ParameterEstimates; ods output InvXpX=InvXpX;

ods output XpX=XpX; run;

proc print data=XpX; title2 "XpX";run;

proc print data=InvXpX; title2 "InvXpX"; run;

proc print data=ParameterEstimates; title2 "Parameter Estimates"; run;

data XpX2; set XpX; n=\_n\_;

data XpX2; set XpX2; where n<3; drop parameter n; title;

proc iml;

use ParameterEstimates;

read all var{Estimate StdErr tValue ProbT} into EstimateMatrix[ colname=vars

];

read all var{Estimate} into EstimateVector; Estimate=t(EstimateVector);

print Estimate;

use XpX2;

read all var{intercept x} into XTX; Cov=Inv(XTX);

contrast={1 -1 0,

0 0 1};

Column={0,0}; Row={1 0 0}; CovarianceMatrix = Column || Cov;

CovarianceMatrix = Row // CovarianceMatrix; prod=contrast\*CovarianceMatrix\*t(contrast);

print XTX, Cov, contrast, CovarianceMatrix, prod; quit;

title 'FiellerM';

%FiellerM(Fieller,2.1-6.990912935,-2.405464142,1.3598686,.1214553,.0484744,16, 1.05499,.05);

title 'FiellerF';

%FiellerF(Fieller,2.1-6.990912935,-2.405464142,1.3598686,.1214553,.0484744,16, 1.05499,.05);

## FiellerM Macro

%macro FiellerM(DataFile,a,b,v11,v12,v22,v,s,alpha);

/\* (C) COPYRIGHT 2001 BY MARY A. MARION \*/

/\* This macro produces a solution to Fieller's equations, that is a (1-alpha)\*100% confidence interval about an estimate of the ratio Mu of two parameters from a simple linear regression function y = alpha + Beta \* x fitted to a simple random sample of size n. Mu, is estimated by a/b.

The errors are assumed to be normally distributed.

Written by Mary A. Marion while a student at Kansas State University April 12,2000.

See class notes pages 8-34. Input Parameters:

DataFile containing x and y (independent and dependent variables)

Regression Parameter Estimates

a = yo- where  = Intercept in regression equation b = Slope from the regression equation

Covariance Matrix of regression parameter estimates V11 Var(a)

V12 Cov(a,b)

V22 Var(b)

v Degrees of freedom = n-2 ( n = number of observations ) s MSE from Simple Linear Regression

alpha Significance level

Acknowledgements:

Draper and Smith (1998), Applied Regression Analysis -Third Edition,

John Wiley & Sons, page 85 (3.28).

Finney, David J. (1978), Statistical Method in Biological Assay -Third Edition, Charles Griffin Company LTD.

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Theory and Application of the Linear Model Wadsworth & Brooks/Cole

Pacific Grove, California

Confidence Interval Formula is given on page 335

Miller, John Ph.D., George Mason University Stat 656 Regression Analysis course notes.

Milliken, George, Ph.D., Kansas State University

STAT 861 Linear Models II course notes, pages 8-34 to 8-37.

SAS Principles of Regression Course Notes, Page 639-640, SAS Institute, Inc.: Cary, North Carolina.

Algorithms are from course notes referenced above. 'Let k = s \* t(alpha/2,v) / b.'

'When 1 - k^2 \* v22 > 0, model assumptions are true.'

'A Confidence Interval Does not Exist if 1- k^2 \* v22 le 0.' 'The Minimum Alpha (observed significance level) is ' stat;

Assumed is a quadratic equation of the form a x^2 + b x + c = 0. a is constrained to be

positive in order for this to work. Hence code includes a test for negative a. \*/

/\* options nomprint nosource; \*/ options formdlim='';

%local AA UnderRad a b v11 v12 v22 v s alpha;

data interval; length msg $ 19; msg=' ';

anum=&a; bnum=&b;

v11=&v11; v12=&v12; v22=&v22; v=&v; alpha=&alpha; s=&s; alpha=&alpha;

data interval; set interval; file print;

stat0 = &b / (&s\*sqrt(v22)); stat0=abs(stat0); stat= 1 - probt(stat0,&v) ; stat=stat\*2;

put / ' The Minimum Alpha (observed significance level) is ' stat; run;

data interval; set interval; coef=1-&alpha/2;

df=&v; t=tinv(coef,df); m=anum/bnum ;

k= (s\*t) / bnum; denominator=1-k\*k\*v22; AA = 1.0 - (k\*k\*v22);

AAI=floor(AA);

call symput('AAI',trim(left(put(AAI,8.))));

B =(2\*k\*k\*v12)-2\*m; C = m\*m- (k\*k\*v11);

UnderRad = B\*\*2 - 4\*AA \* C;

call symput('UnderRad',trim(left(put(UnderRad,16.8)))); run;

%if %eval(&AAI) < 0 %then %do;

/\* The if statement checks to see whether 1-k\*k\*v22 is le 0. This is same as seeing if AAI < 0 \*/

data interval; set interval; msg = 'Assumptions not Met' ;

options formdlim=' '; title1 ' '; proc print data=interval noobs label; title2 "Interval Diagnostics";

label UnderRad='Under Radical' denominator='1-k^2\*v22' AA='A'; run; title2;

%end; %else %do;

data interval; set interval; if AA lt 0 then do;

AA = -AA; B = -B; C=-C; end;

LB = ( -B - sqrt(B\*\*2-4\*AA\*C) ) / (2\*AA); UB = ( -B + sqrt(B\*\*2-4\*AA\*C) ) / (2\*AA); msg='Assumptions are Met'; run;

options formdlim=' ';

proc print data=interval noobs label; var anum bnum df alpha t LB m UB ; label anum='a' bnum='b'

m=ratio LB='Lower Bound' UB='Upper Bound' t='Critical Value from t-distribution'; run;

proc print data=interval noobs label; title2 "Interval Details"; label UnderRad='Under Radical' denominator='1-k^2\*v22' AA='A'; run; title2;

%end;

title2;

proc datasets; delete interval; run &cancel; options mprint source;

%mend FiellerM;

## FiellerF macro

%macro FiellerF(DataFile,a,b,v11,v12,v22,v,s,alpha);

/\* (C) COPYRIGHT 2001 BY MARY A. MARION \*/

/\* This macro produces a solution to Fieller's equations, that is a (1-alpha)\*100% confidence interval about an estimate of the ratio Mu of two parameters from a simple linear regression function y = alpha + Beta \* x fitted to a simple random sample of size n. Mu, is estimated by a/b.

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a = yo- where  = Intercept in regression equation b = Slope from the regression equation

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V12 Cov(a,b)

V22 Var(b)

v Degrees of freedom = n-2 ( n = number of observations ) s MSE from Simple Linear Regression

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Algorithms are from course notes referenced above. 'Let k = s \* t(alpha/2,v) / b.'

'When 1 - k^2 \* v22 > 0, model assumptions are true.'

'A Confidence Interval Does not Exist if 1- k^2 \* v22 le 0.' 'The Minimum Alpha (observed significance level) is ' stat;

/\* options nomprint nosource; \*/ options formdlim='';

%local a b v11 v12 v22 v s alpha;

data interval; length msg $ 19; msg=' ';

anum=&a; bnum=&b;

v11=&v11; v12=&v12; v22=&v22; v=&v; alpha=&alpha; s=&s; alpha=&alpha;

coef=1-&alpha/(2); df=&v;

data interval; set interval; file print;

stat0 = &b / (&s\*sqrt(v22)); stat0=abs(stat0);

stat= 1 - probt(stat0,&v) ; stat=stat\*2;

put / ' The Minimum Alpha (observed significance level) is ' stat; run;

data interval; set interval;

/\* Equation 4.12.3 \*/ t=tinv(coef,df);

g = t\*t \* s\*s \* v22 / (bnum\*bnum);

call symput('g',trim(left(put(g,15.8)))); m=anum/bnum ;

/\* Equation 4.12.3 \*/

disc=(v11-2\*m\*v12+m\*m\*v22-g\*(v11-(v12\*\*2)/v22));

if disc<0 then code\_disc='NO '; else code\_disc="YES"; if disc<0 then k=.; else k=sqrt(disc);

dif= ( (t\*s) / ( abs(bnum) ) ) \* k ; val=m-g\*v12/v22;

LB=(val-dif)/(1-g);

UB=(val+dif)/(1-g);

if LB>UB then do;

dummy=LB; LB=UB; UB=dummy; end;

code\_g='YES'; msg='Assumptions are Met'; if g ge 1 then do;

code\_g='NO '; msg = 'Assumptions not Met' ; end; if g ge 1 and disc<0 then LB=.;

if g ge 1 and disc<0 then UB=.;

options formdlim=' '; title;

proc print data=interval noobs label; var anum bnum df alpha g LB m UB ; label anum='a' bnum='b'

m=ratio LB='Lower Bound' UB='Upper Bound' t='Critical Value from t-distribution'; run;

proc print data=interval noobs label; title2 "Interval Details";

label anum='a' bnum='b';

var anum bnum m s v11 v12 v22 df code\_disc code\_g LB UB; run; title2;

proc datasets; delete interval; run cancel; options mprint source;

%mend FiellerF;

## REGINV2 macro for restricted calibration problems.

%macro reginv2(olddata,x,y,alpha,resids);

/\* This macro produces inverse confidence intervals for individual values for the simple linear regression problem found in dataset olddata.

Text: SAS Principles of Regression Course Notes, Page 639-640 Text: Draper and Smith (1998),

Applied Regression Analysis -Third Edition, page 85 (3.28)

Text: Graybill, Franklin A. (1976),

Theory and Application of the Linear Model \*/ options nomprint nosource;

proc reg data=&olddata outest=estimate (keep=\_rmse\_ &x intercept rename=(&x=b1 intercept=b0));

model &y=&x /covb; output out=&resids

p=p r=r press=press l95=l95 u95=u95

l95m=l95m u95m=u95m

stdp=stdp stdi=stdi stdr=stdr h=h cookd=cookd covratio=covratio dffits=dffits

student=student rstudent=rstudent; run;

proc contents data=estimate; title2 'estimate'; run cancel; proc print data=estimate; title2 'estimate'; run cancel; proc means noprint data=&olddata;

var &x;

output out=means(keep=xmean n css) mean=xmean n=n css=css;

run;

/\* proc print data=means; title 'means'; run; \*/ data combine(drop=\_rmse\_);

set estimate; set means; mse=\_rmse\_\*\_rmse\_;

/\* proc print data=combine;

title1 'Approximate Confidence Intervals'; title2 'Summary Information for Calculations'; title3 'combine';

run; \*/

data final(drop=&x);

if \_n\_=1 then set combine; set &olddata;

if &x=.;

/\* Graybill, page 277 (8.5.5) \*/ xpred=(&y-b0)/b1;

/\* proc print data=final; run; \*/ data interval; set final;

a1=1/n;

diff=(xpred-xmean)\*\*2; a2=diff/css; mult=mse/b1\*\*2; var=mult\*(1+a1+a2); stderr=sqrt(var);

df=n-2;

coef=1-&alpha/2; t=tinv(coef,df); upper=xpred+t\*stderr; lower=xpred-t\*stderr; g=(mse\*t\*\*2)/(css\*b1\*\*2); alpha=&alpha;

proc print data=interval noobs label;

var &y lower xpred upper alpha stderr t g; label &y = 'Value of Dependent Variable'

upper='Upper Bound' lower='Lower Bound'

xpred='Predicted Value of Independent Variable'

alpha='Value of Alpha'

stderr='Standard Error of Independent Variable' t='Critical Value from t-distribution';

title3 'CALIBRATION';

title4 'Value of G Should be Less than 0.1'; run;

title3; title4;

proc datasets; delete combine estimate final; run; options mprint source;

%mend reginv2;

## Program to run reginv2 for use with restricted calibration problems.

/\* Graybill, Linear Models Problem 8.10, page 336, data is on page 274 \*/ options nodate number center linesize=76 pagesize=54 pageno=1;

options replace probsig=1 firstobs=1 obs=max; options mrecall label;

options xsync noxwait;

options mlogic mprint symbolgen formdlim='' label;

data olddata;

infile "c:\Fieller.dat"; input x y;

data addin; input x y; datalines;

. 2.10

;

data olddata; set olddata addin; run; proc sort; by x; run;

proc print;run;

%reginv2(olddata,x,y,.05,resids);

# SAS OUTPUT

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **GRAYBILL.8.10.LST** | Obs | x | y | 1 |
|  | 1 | . | 2.10 |  |
|  | 2 | 1.0 | 4.81 |  |
|  | 3 | 1.1 | 3.60 |  |
|  | 4 | 1.3 | 4.90 |  |
|  | 5 | 1.6 | 3.05 |  |
|  | 6 | 1.8 | 3.44 |  |
|  | 7 | 1.8 | 3.17 |  |
|  | 8 | 1.8 | 3.34 |  |
|  | 9 | 2.1 | 1.61 |  |
|  | 10 | 2.4 | 1.22 |  |
|  | 11 | 2.6 | 0.20 |  |
|  | 12 | 2.6 | 1.56 |  |
|  | 13 | 2.7 | 0.55 |  |
|  | 14 | 2.9 | -2.56 |  |
|  | 15 | 3.0 | -0.34 |  |
|  | 16 | 3.5 | -2.56 |  |
|  | 17 | 3.6 | -2.96 |  |
|  | 18 | 4.1 | -1.04 |  |
|  | 19 | 5.2 | -4.64 |  |
|  |  |  |  | 2 |

The GLM Procedure Number of observations 19

NOTE: Due to missing values, only 18 observations can be used in this analysis.

3

The GLM Procedure

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | The X'X  Intercept | Matrix | | x | y |  |
| Intercept | 18 |  | | 45.1 | 17.35 |
| x | 45.1 |  | | 133.63 | -6.152 |
| y | 17.35 |  | | -6.152 | 153.8989 |
|  |  |  | |  |  | 4 |
|  | The GLM | Procedure | |  |  |  |
|  | X'X Inverse | | Matrix | |  | |
|  | Intercept | | x | | y | |
| Intercept | 0.3598685805 | | -0.121455309 | | 6.9909129346 | |
| x | -0.121455309 | | 0.0484744028 | | -2.405464142 | |
| y | 6.9909129346 | | -2.405464142 | | 17.808145181 | |

5

The GLM Procedure

Dependent Variable: y

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | DF | Sum of  Squares | Mean Square | F Value | Pr > F |
| Model | 1 | 119.3672826 | 119.3672826 | 107.25 | <.0001 |
| Error | 16 | 17.8081452 | 1.1130091 |  |  |
| Corrected Total | 17 | 137.1754278 |  |  |  |
| R-Square | Coeff | Var Root | MSE y | Mean |  |

0.870180 109.4517 1.054992 0.963889

Source DF Type III SS Mean Square F Value Pr > F

x 1 119.3672826 119.3672826 107.25 <.0001

Standard

Parameter Estimate Error t Value Pr > |t|

Intercept 6.990912935 0.63287992 11.05 <.0001

x -2.405464142 0.23227667 -10.36 <.0001

6

XpX

Obs Parameter Intercept x y 1 Intercept 18 45.1 17.35

2 x 45.1 133.63 -6.152

3 y 17.35 -6.152 153.8989

7

InvXpX

Obs Parameter Intercept x y 1 Intercept 0.3598685805 -0.121455309 6.9909129346

2 x -0.121455309 0.0484744028 -2.405464142

3 y 6.9909129346 -2.405464142 17.808145181

8

Parameter Estimates

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Obs | Dependent | Parameter | Estimate | StdErr | tValue | Probt |
| 1 | y | Intercept | 6.990912935 | 0.63287992 | 11.05 | <.0001 |
| 2 | y | x | -2.405464142 | 0.23227667 | -10.36 | <.0001 |

9

ESTIMATE 6.9909129 -2.405464

XTX

18 45.1

45.1 133.63

COV 0.3598686 -0.121455

-0.121455 0.0484744

|  |  |  |
| --- | --- | --- |
|  | CONTRAST |  |
| 1 | -1 | 0 |
| 0 | 0 | 1 |

COVARIANCEMATRIX

|  |  |  |
| --- | --- | --- |
| 1 | 0 | 0 |
| 0 | 0.3598686 | -0.121455 |
| 0 | -0.121455 | 0.0484744 |

PROD 1.3598686 0.1214553

0.1214553 0.0484744

FiellerM 10

The Minimum Alpha (observed significance level) is 1.6845455E-8

FiellerM 11

Critical Value

from Lower Upper a b df alpha t-distribution Bound ratio Bound

-4.89091 -2.40546 16 0.05 2.11991 1.03150 2.03325 2.99369

FiellerM 12

Interval Details

msg anum bnum v11 v12 v22 v

Assumptions are Met -4.89091 -2.40546 1.35987 0.12146 0.048474 16

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| alpha | s | stat0 | | stat | coef | df | t | m | k |
| 0.05 1.05499 | | 10.3561 | | 1.6845E-8 | 0.975 | 16 | 2.11991 | 2.03325 | -0.92975 |
| 1-k^2\* v22 | A | | AAI | B | C | | Under Radical | LB | UB |
| 0.95810 | 0.95810 | | 0 | -3.85652 | 2.95859 | | 3.53428 | 1.03150 | 2.99369 |

FiellerF 13

The Minimum Alpha (observed significance level) is 1.6845455E-8

14

Lower Upper

a b df alpha g Bound ratio Bound

-4.89091 -2.40546 16 0.05 0.041903 1.03150 2.03325 2.99369

15

Interval Details

1

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| a  -4.89091 | b  -2.40546 | m  2.03325 | s  1.05499 | v11  1.35987 | v12  0.12146 |
| v22 | code\_  df disc | code\_g | LB | UB |  |
| 0.048474 | 16 YES | YES | 1.03150 | 2.99369 |  |
| **REGINV.8.10.LST** |  |  |  |  |  |
|  |  | The SAS | System |  |  |
| Obs x | | | y | | |
| 1 . | | | 2.10 | | |
| 2 1.0 | | | 4.81 | | |
| 3 1.1 | | | 3.60 | | |
| 4 1.3 | | | 4.90 | | |
| 5 1.6 | | | 3.05 | | |
| 6 1.8 | | | 3.44 | | |
| 7 1.8 | | | 3.17 | | |
| 8 1.8 | | | 3.34 | | |
| 9 2.1 | | | 1.61 | | |
| 10 2.4 | | | 1.22 | | |
| 11 2.6 | | | 0.20 | | |
| 12 2.6 | | | 1.56 | | |
| 13 2.7 | | | 0.55 | | |

|  |  |  |  |
| --- | --- | --- | --- |
| 14 | 2.9 | -2.56 |  |
| 15 | 3.0 | -0.34 |  |
| 16 | 3.5 | -2.56 |  |
| 17 | 3.6 | -2.96 |  |
| 18 | 4.1 | -1.04 |  |
| 19 | 5.2 | -4.64 |  |
|  | The SAS | System | 2 |

The REG Procedure Model: MODEL1 Dependent Variable: y

Analysis of Variance

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Source | DF | Sum of  Squares | Mean  Square | F Value | Pr > F |
| Model | 1 | 119.36728 | 119.36728 | 107.25 | <.0001 |
| Error | 16 | 17.80815 | 1.11301 |  |  |
| Corrected Total | 17 | 137.17543 |  |  |  |
| Root MSE |  | 1.05499 | R-Square | 0.8702 |  |
| Dependent | Mean | 0.96389 | Adj R-Sq | 0.8621 |  |
| Coeff Var |  | 109.45167 |  |  |  |

Parameter Estimates Parameter Standard

Variable DF Estimate Error t Value Pr > |t|

Intercept 1 6.99091 0.63288 11.05 <.0001

x 1 -2.40546 0.23228 -10.36 <.0001

Covariance of Estimates

Variable Intercept x Intercept 0.4005369955 -0.135180861

x -0.135180861 0.0539524502

The SAS System 3

estimate CALIBRATION

Value of G Should be Less than 0.1

Predicted

Value of Value of

Dependent Lower Independent Upper Value of

|  |  |  |
| --- | --- | --- |
| Variable | Bound Variable Bound | Alpha |
| 2.1 | 1.07314 2.03325 2.99336 | 0.05 |
| Standard Error of | Critical Value |  |
| Independent Variable | from  t-distribution g |  |
| 0.45290 | 2.11991 0.041903 |  |